A Slacks-based Measure of Super-efficiency in DEA with Interval Data

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Abstract: Data envelopment analysis (DEA) is a non-parametric technique for measuring the efficiency of decision making units (DMU) with exact input and output terms. So far, a number of DEA models with interval data have been developed. The radial DEA model such as CCR or BCC model with interval data are well known as basic DEA models with interval data. However, these radial DEA models have some defects. In this study, we develop a new approach based upon the slacks-based measurement (SBM) of super-efficiency model for dealing with interval data in DEA. The SBM super-efficiency DEA model can not only discriminate the performance of efficient DMUs from inefficient ones, but can distinguish between the efficient DMUs. In addition, this model can overcome the drawback that the super-efficiency model can be infeasible under variable returns to scale (VRS) assumption. An illustrative example is provided to verify the idea of this paper and show its potential application and validity.

Keywords: data envelopment analysis, interval efficiency, super-efficiency, slacks, interval data

1 Introduction

Data envelopment analysis (DEA), originally developed by Charnels et al. in 1978[1], is a nonparametric methodology for measuring and evaluating the relative efficiency of decision making units (DMUs) with multiple incommensurate inputs and outputs. With the in-depth study of the DEA methods, more and more domestic and foreign scholars began to discuss the DEA theory and its applications with uncertainty. It is generally known that the original DEA models assume all input and output data are deterministic. However, there exists a mass of uncertainties in real economic and production management activities such as measurement error, data noisy, incomplete information and randomness of economic phenomena and laws and so on. Therefore, it is worthwhile to enrich the DEA theory and methodology and its real applications in uncertain environment.

In order to deal with the uncertain optimization problems, fuzzy and stochastic DEA approaches are commonly used to describe the imprecise characteristics. In stochastic DEA model[2-3], the uncertain coefficients are regarded as random variables and their probability distributions are assumed to be known. In fuzzy DEA model[4-6], the constraints and objective function are viewed as fuzzy sets and their membership functions also need to be known. In those two kinds of approaches, the membership functions and probability distributions play important roles. However, it is sometimes difficult to specify an appropriate membership function or accurate probability distribution in an uncertain environment[7]. In recent years, the interval analysis method was developed to model the uncertainty in uncertain optimization problems, in which the bounds of the uncertain coefficients are only required, not necessarily knowing the probability distributions or membership functions. Cooper, Park and Yu[8] was the first to discuss how to deal with imprecise data such as bounded data, ordinal data, and ratio bounded data. Some factors such as labor force employed and used in an evaluation period can be given its upper and lower bound. It is fluctuating due to the frequent leaving and entering in such small business units. For some qualitative factors such as professional quality, service attitude, level of management and so on, can be attributed to ratio bounds or ordinal data, as discussed in Lee et al.[9]. If we incorporate these imprecise data into the classic DEA model, the resulting DEA model is a non-linear and non-convex program, which can be transformed into a linear problem equivalent by adopting scale transformations and variable alternations on the data[10-12]. Zhu[13] simplified their method to reduce the computational burden. The efficiency score is a real number.

Different from the method discussed above, Despotis and Smirlis[14] developed an alternative method for solving DEA model with interval data. They transform the non-linear DEA model into a linear programming equivalent, on the basis of the original data set, by applying transformations only on the variables. The resulting efficiency scores is an interval numbers and many scholars call it as interval efficiency. Subsequently, based on this idea, Entani et al.[15] formulated a DEA model with an interval efficiency which consists of efficiencies obtained from the
optimistic and pessimistic viewpoints and extend it to interval data and fuzzy data. But there is a drawback with their model. To overcome this defect, Wang et al.\cite{16} develop a new pair of interval DEA models for interval input and output data rather than crisp input and output data. The interval efficiency scores will be obtained by the best lower bound efficiency and the best upper bound efficiency. Haghighat and Khorram\cite{17} discussed the problem of maximum and minimum numbers of DEA efficient units. Jahanshahloo et al.\cite{18} consider FDH model with imprecise data. Jahanshahloo et al.\cite{19} proposed a generalized model with interval data for interval DEA (IDEA), called IGDEA model, which can treat basic IDEA models, specifically the CCR, BCC, FDH models with interval data in a unified way. Esmaeili\cite{20} develop a new approach based upon the Enhanced Russell Measuer (ERM) for dealing with interval data in DEA. As we all know, DMUs are improved so that their lower bounds become as large as to attain the maximum value one. The points obtained by this method are referred as ideal points proposed in Jahanshahloo et al.\cite{21}. In their article, they rank DMUs by ideal points for each DMU and extend the ranking model to interval data. Jahanshahloo et al.\cite{22}and Yang et al.\cite{23}extended a cross-efficiency model for ranking units into interval data. Jahanshahloo et al.\cite{24}presented a new TOPSIS approach for ranking DMUs with interval data yielding the interval efficiency scores for each alternative. To sum up, most of the literatures mentioned above are on the basis of radial DEA model such as CCR or BCC model or their dual models. These basic DEA models can distinguish the performance of efficient DMUs from inefficient ones, but they lack the power to discriminate between efficient units.

The main purpose of this paper is to investigate the interval efficiency analysis in non-radial DEA model. We consider a slacks-based measure (SBM) of super-efficiency DEA model developed by Tone\cite{25} with interval data. It can discriminate between efficient DMUs. Our method can rank DMUs that belong to $E^{++}$, i.e. the lower and upper bound of interval efficiencies are both more than unity. In addition, it is well known that the super-efficiency DEA approach can be infeasible under the condition of variable returns to scale (VRS)\cite{25}. However, the proposed DEA approach can overcome the above-mentioned problems, that is, the slacks-based measure of super-efficiency model is always feasible under constant or variable returns to scale assumption\cite{27}.

The rest of this paper is organized as follows. In Section 2, we provide preliminary information that will be used in the succeeding sections. Section 3 reviews super-efficiency with interval data from analytical and computational perspectives. Section 4 is the main part of this paper where we formulate a super SBM model for dealing with interval data. Then we can get the upper and lower bounds of efficient scores for each DMU. Also, we will categorize DMUs based upon their interval efficiency scores. In section 5, we introduce an approach for comparing and ranking interval efficiencies of DMUs.

Section 6 contains an illustrative example. The last section concludes the paper with the summary and conclusions.

2 Preliminaries

Suppose that there are $n$ DMUs for evaluation and each DMU produce $s$ outputs by consuming $m$ inputs. All output and input data are assumed to be non-negative, and each DMU has at least one strictly positive input and output. The following notation will be used throughout this paper.

Nomenclature

$DMU_j$ is the $j$th decision making unit, $DMU_{h}$ is the decision making unit under evaluation, $x_{j} = (x_{i1}, x_{i2}, \cdots, x_{in})^{T} \in \mathbb{R}_{0}^{nx1}$ is the column vector of inputs consumed by $DMU_j$, $y_{j} = (y_{j1}, y_{j2}, \cdots, y_{js})^{T} \in \mathbb{R}_{0}^{sx1}$ is the column vector of outputs produced by $DMU_j$, $y_{j0} = (y_{j01}, y_{j02}, \cdots, y_{j0s})^{T} \in \mathbb{R}_{0}^{sx1}$ is the column vector of outputs produced by $DMU_j$.

$y_{j0} = (y_{j01}, y_{j02}, \cdots, y_{j0s})^{T} \in \mathbb{R}_{0}^{sx1}$ is the column vector of outputs produced by $DMU_j$.

The main purpose of this paper is to investigate the interval efficiency analysis in non-radial DEA model. We consider a slacks-based measure (SBM) of super-efficiency DEA model developed by Tone\cite{25} with interval data. It can discriminate between efficient DMUs. Our method can rank DMUs that belong to $E^{++}$, i.e. the lower and upper bound of interval efficiencies are both more than unity. In addition, it is well known that these two models have the same optimal solutions.
\[ \theta^i = \min \theta \]
\[
\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x^{i}_{ji} \leq \theta x^{i*}_{ji}, \quad i \in N_m
\]
\[ \text{(1)} \]
\[
\sum_{j=1}^{n} \lambda_j y^{i}_{jr} \geq y^{i*}_{jr}, \quad r \in N_s,
\]
\[
\lambda_j \in \Delta, \quad j \in N_s
\]
In order to deal with such an uncertain situation, the following pair of LP models has been developed to generate the upper and lower bounds of interval efficiency for each DMU:
\[ \theta^*_i = \min \theta \]
\[
\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x^{i}_{ji} \leq \theta x^{i*}_{ji}, \quad i \in N_m
\]
\[ \text{(2)} \]
\[
\sum_{j=1}^{n} \lambda_j y^{i}_{jr} \geq y^{i*}_{jr}, \quad r \in N_s
\]
and
\[ \theta^{*-}_i = \min \theta \]
\[
\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x^{i*}_{ji} \leq \theta x^{i*}_{ji}, \quad i \in N_m
\]
\[ \text{(3)} \]
\[
\sum_{j=1}^{n} \lambda_j y^{i*}_{jr} \geq y^{i*}_{jr}, \quad r \in N_s
\]
\[
\lambda_j \in \Delta, \quad j \in N_s
\]
After solving the models (2) and (3), the interval super-efficiency scores are denoted by \( \theta^*_{ij} = [\theta^*_{ij}, \theta^{*-}_{ij}] \) and DMUs are divided into three classes:
Class 1: Include all DMUs which are super-efficient both in their best and worst situation; in other words,
\[ E^+ = \{DMU_{ij}, j \in N_s | \delta^*_j \geq 1 \} \]
Class 2: Consists of all DMUs which are efficient in their best situation, but inefficient in their worst situation; in other words,
\[ E^- = \{DMU_{ij}, j \in N_s | \delta^*_j < 1 \text{ and } \delta^{*-}_j \geq 1 \} \]
and
Class 3: Consists of all DMUs which are inefficient in their best situation. It goes without saying that such DMUs are also inefficient in their worst situation; in other words,
\[ E^- = \{DMU_{ij}, j \in N_s | \delta^*_j < 1 \} \]

4 The SBM super-efficiency DEA model with interval data

Based on the above discussions, we can be certain that the Interval super-efficiency can be a good evaluation tool for dealing with interval data, but it may fail to provide feasible solutions under VRS condition. As such, we propose the use of SBM super-efficiency DEA model with interval data. Following Tone’s concept[26], we can assume a production possibility set
\[ P \setminus \{x_{ji}, y_{jr} \} \text{ covering } (X, Y) \text{ but excluding } (x_{ji}, y_{jr}) \]. Due to the existence of uncertainty, the assumption that input and output data in super SBM model are known exactly may not be true. They are only known to lie within bounded intervals \( x^{i*}_{ji} = [x^{i*1}_{ji}, x^{i*2}_{ji}], y^{i*}_{jr} = [y^{i*1}_{jr}, y^{i*2}_{jr}] \) and \( y^{i*}_{jr} = [y^{i*1}_{jr}, y^{i*2}_{jr}] \), where \( x^{i*}_{ji} > 0 \) and \( y^{i*}_{jr} > 0 \). The resulting SBM super-efficiency score can be an interval.

4.1 Model formulation

\[ \min \delta^*_i = \frac{1}{s} \sum_{j=1}^{n} \frac{\bar{x}^{i*}_{ji}}{x_{ji}}, \frac{1}{s} \sum_{j=1}^{n} \frac{\bar{y}^{i*}_{jr}}{y_{jr}} \]
\[
\text{s.t.} \quad \bar{x}_{ji} = \sum_{j=1}^{n} \mu_j x^{i*}_{ji}, \quad i \in N_m
\]
\[ \bar{y}_{jr} = \sum_{j=1}^{n} \mu_j y^{i*}_{jr}, \quad r \in N_s
\]
\[ \mu_j \in \Delta, \quad j \in N_s \]

Since the modality of interval programming is similar to the modality of multi-objective program, the DEA model with interval data can be dealt with by solving a series of linear programs with deterministic data for each unit. Therefore, all of efficiency scores can establish an interval number. According to the interval arithmetic on objective function, we have
\[ \min \delta^*_i = \frac{1}{s} \sum_{j=1}^{n} \frac{\bar{x}^{i*}_{ji}}{x_{ji}}, \frac{1}{s} \sum_{j=1}^{n} \frac{\bar{y}^{i*}_{jr}}{y_{jr}} \]
\[ \text{s.t.} \quad \bar{x}_{ji} = \sum_{j=1}^{n} \mu_j x^{i*}_{ji}, \quad i \in N_m
\]
\[ \bar{y}_{jr} = \sum_{j=1}^{n} \mu_j y^{i*}_{jr}, \quad r \in N_s
\]
\[ \mu_j \in \Delta, \quad j \in N_s \]

It is apparently that \( \delta^*_i \) should be an interval number, which we can denote by \( [\delta^*_i, \delta^{*-}_i] \).

Therefore, the objective function takes the form
\[ \delta^*_i = [\delta^*_i, \delta^{*-}_i] = \frac{1}{s} \sum_{j=1}^{n} \frac{\bar{x}^{i*}_{ji}}{x_{ji}}, \frac{1}{s} \sum_{j=1}^{n} \frac{\bar{y}^{i*}_{jr}}{y_{jr}} \]

Model (4) is a non-linear and non-convex programming. The SBM super-efficiency score obtained by \( DMU_{ij} \) in Model (4) is not worse (less) than any other SBM super-efficiency score that the DMU might attain,
by adjusting the levels of the inputs and outputs within the limits of the bounded intervals. The Model (4) is under an interval data setting, all output and input data are intervals and the SBM super-efficiency score is also an interval number. The upper and lower bounds of the relative efficiency of DMU, are obtained by the following pair of fractional programming for respectively:

\[
\min \delta_h^i = \frac{\sum_{r=1}^{n} \omega_r y_{rj}^v}{\sum_{r=1}^{n} \omega_r y_{rj}^u} \\
\text{s.t.} \quad \bar{x}_i \geq \sum_{j=1}^{n} \mu_j x_{ij} \quad i \in N_m
\]

\[
\bar{y}_r \leq \sum_{i=1}^{n} \mu_j y_{rij} \quad r \in N_s
\]

\[
\bar{x}_i \geq \mu_j x_{ij} \quad i \in N_m
\]

\[
0 \leq \bar{y}_r \leq y_{rj}^u \quad r \in N_s
\]

\[
\mu_j \in \Delta \quad j \in N_n
\]

Model (5) is a DEA model with exact data, where the levels of inputs and outputs are adjusted unfavorably the evaluated unit DMU, and in favor of the other units DMU. For the evaluated unit DMU, the inputs are adjusted at the upper bounds and the outputs at the lower bounds. For the other units DMU, the inputs are favorably adjusted at their lower bounds and the outputs at their upper bounds. Thus we can get the efficiency attained by DMU, in model (6) serves as a lower bound of its possible efficiency scores. 

Using Charnes-Cooper transformation[30], the above pair of fractional programming models (5) and (6) can be simplified as the following equivalent linear programming models (7) and (8):

\[
\min \tau_h^i = \frac{\sum_{r=1}^{n} \omega_r y_{rj}^v}{\sum_{r=1}^{n} \omega_r y_{rj}^u} \\
\text{s.t.} \quad \sum_{s=1}^{n} \omega_r y_{rj}^u = 1 \quad r \in N_s
\]

\[
\sum_{s=1}^{n} \omega_r x_{ij} \quad i \in N_m
\]

\[
\bar{y}_r \leq \sum_{i=1}^{n} \omega_r y_{rij} \quad r \in N_s
\]

\[
\bar{x}_i \geq x_{ij} \quad i \in N_m
\]

\[
0 \leq \bar{y}_r \leq y_{rj}^u \quad r \in N_s
\]

\[
\omega_j \in \Lambda \quad j \in N_j
\]

and

\[
\min \tau_h^i = \frac{\sum_{r=1}^{n} \omega r y_{rj}^v}{\sum_{r=1}^{n} \omega r y_{rj}^u} \\
\text{s.t.} \quad \sum_{s=1}^{n} \omega_r y_{rj}^u = 1 \quad r \in N_s
\]

\[
\sum_{s=1}^{n} \omega_r x_{ij} \quad i \in N_m
\]

\[
\bar{y}_r \leq \sum_{i=1}^{n} \omega_r y_{rij} \quad r \in N_s
\]

\[
\bar{x}_i \geq x_{ij} \quad i \in N_m
\]

\[
0 \leq \bar{y}_r \leq y_{rj}^u \quad r \in N_s
\]

\[
\omega_j \in \Lambda \quad j \in N_j
\]

where the SBM super-efficiency \( \tau_h^i \) stands for the upper bound of the best possible relative efficiency of DMU, when all the DMUs are in the state of best production activity. While the SBM super-efficiency \( \tau_h^i \) stands for the lower bound of the best possible relative efficiency of DMU, .

4.2 Classification and discrimination of the units

Based on the idea of the previous classification method in Jahanshahloo et al.[19] and considering the obtained interval SBM super-efficiency of any DMU lies
in an interval, all DMUs can be divided into one of the three following classes:

Class 4: Include all DMUs which are efficient both in their best and worst situation; in other words,

\[ E^+ = \{ DMU_j, j \in N_n | \delta_j^* > 1 \} \]

Class 5: Consists of all DMUs which are efficient in their best situation, but inefficient in their worst situation; in other words,

\[ E^* = \{ DMU_j, j \in N_n | \delta_j^* = 1 and \delta_j^1 > 1 \} \]

and

Class 6: Consists of all DMUs which are inefficient in their best situation. It goes without saying that such DMUs are, also, inefficient in their worst situation; in other words,

\[ E^- = \{ DMU_j, j \in N_n | \delta_j^1 = \delta_j^* = 1 \} \]

5 Comparing and ranking interval efficiencies

Now let’s talk about the method of comparing and ranking any two interval efficiencies. Zhang et al. (1999)[33] developed a method for comparing intervals based on three relations of two intervals on real line. Based on their idea, Jiang et al. (2008)[32] modified possibility degree on the basis of all of the possible relations between two intervals. In essence, these two methods give the same results. Here, we use the former method. According to what has been discussed above, each efficient score of DMUs is included in \( \delta_j^1 = [\delta_j^*, \delta_j^] \), and precisely speaking, dividing interval \( \delta_j^1 = [\delta_j^*, \delta_j^] \) into \( n \) equivalent parts, we can obtain \( n \) small intervals which one of them cover the efficient score identically, in this way, it is assumed that all interval efficient scores are regarded as random variables and follow uniform distributions in \( \delta_j^1 = [\delta_j^*, \delta_j^] \). For convenience sake, we first consider there are only two DMUs, i.e. \( DMU_j \) and \( DMU_j' \), their efficient scores are individually defined as \( \delta_j^1 = [\delta_j^*, \delta_j^] \) and \( \delta_j'^1 = [\delta_j'^*, \delta_j'^] \). The following definition of possibility degree based on three relations of interval numbers on the real number axis is presented:

Definition 1 let \( \delta_j^1 = [\delta_j^*, \delta_j^] \) and \( \delta_j'^1 = [\delta_j'^*, \delta_j'^] \) where \( \delta_j^1 \neq \delta_j'^1 \) and \( \delta_j^1 > \delta_j'^1 \), the degree of possibility of the ‘interval \( \delta_j^1 \) to be superior to the interval \( \delta_j'^1 \) is defined by the following formula:

\[ P_{\delta_j^1 \sim \delta_j'^1} = \begin{cases} 0 & \delta_j^1 \geq \delta_j'^1 \\ 0.5 \frac{\delta_j^* - \delta_j^1 + \delta_j'^1 - \delta_j'^*}{\delta_j^* - \delta_j^1 + \delta_j'^1 - \delta_j'^*} & \delta_j^1 \leq \delta_j'^1 \leq \delta_j^* \\ \frac{\delta_j^1 - \delta_j'^1}{\delta_j^* - \delta_j^1} + 0.5 \frac{\delta_j'^* - \delta_j^1}{\delta_j'^* - \delta_j'^1} & \delta_j^* < \delta_j'^1 \leq \delta_j^* \\ \frac{\delta_j^1 - \delta_j'^1}{\delta_j^* - \delta_j^1} + 0.5 \frac{\delta_j^1 - \delta_j'^1}{\delta_j^1 - \delta_j'^1} & \delta_j^1 < \delta_j'^1 \leq \delta_j^* \\ \end{cases} \]  \hspace{1cm} (9)

where the \( P_{\delta_j^1 \sim \delta_j'^1} \) or \( P_{\delta_j'^1 \sim \delta_j^1} \) is regarded as the probability for random variable \( \delta_j^1 \) larger or smaller than \( \delta_j'^1 \) and the symbol ‘\( \sim \)’ represents ‘be superior to’.

Theorem 1 When \( \delta_j^1 > \delta_j'^1 \), then \( P_{\delta_j^1 \sim \delta_j'^1} + P_{\delta_j'^1 \sim \delta_j^1} = 1 \).

Through the above computation, a probability degree matrix of any two interval efficient scores of all DMUs is constructed, that is \( P = \begin{bmatrix} 0.5 & P_{12} & \cdots & P_{1n} \\ P_{21} & 0.5 & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & 0.5 \end{bmatrix} \), where \( P_j \) denotes the possibility for \( \delta_j^1 \) larger than \( \delta_j'^1 \). Due to the transitivity of possibility degree[33], the interval number becomes more superior with the increase of the possibility degree. For multiple interval number comparisons, the ranking result can be obtained by the following formula[33], \( D_j = \sum_{k=1}^{n} P_{jk} \cdot j \in N_n \).

Theorem 2 Let \( P = (P_{xy})_{n \times n} \) be the possibility degree matrix established by paired comparison of interval efficiencies. If \( \delta_j^1 > \delta_j'^1 (\delta_j^1 \leq \delta_j'^1) \), then \( D_j \geq D_{j'} \) \((D_j \leq D_{j'})\).

Therefore, the ordering vector is defined as \( D = (D_1, D_2, \cdots, D_n)^T \), we can acquire the ranking of all DMUs by comparing elements \( D_j \) \((j \in N_n)\) in \( D \).

6 An application to enterprise project evaluation

Considering a performance assessment problem of self-innovation projects in an enterprise of Shenzhen City in China where five projects (DMUs) are to be evaluated in terms of three inputs and one output. In each project, the number of participants (NOP), gross investment (GI) and energy consumption (EC) are considered as inputs. Annual production (AP) is considered as output. The data about the inputs and outputs are uncertain due to the availability and forecast error and therefore estimated as interval numbers. Tab.1 shows the input and output data for the five projects of an enterprise. Models are solved by using Lingo 11.0 software.
Tab.1 Interval data for 5 DMUs with 3 inputs and 1 output

<table>
<thead>
<tr>
<th>Projects (DMUs)</th>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NOP</td>
<td>GI</td>
</tr>
<tr>
<td>j</td>
<td>$x_{ij}^l$</td>
<td>$x_{ij}^u$</td>
</tr>
<tr>
<td>D1</td>
<td>95</td>
<td>105</td>
</tr>
<tr>
<td>D2</td>
<td>95</td>
<td>105</td>
</tr>
<tr>
<td>D3</td>
<td>175</td>
<td>185</td>
</tr>
<tr>
<td>D4</td>
<td>115</td>
<td>125</td>
</tr>
<tr>
<td>D5</td>
<td>195</td>
<td>205</td>
</tr>
</tbody>
</table>

Tab.2 Interval efficiencies of the five projects using models (2) and (3)

<table>
<thead>
<tr>
<th>Projects</th>
<th>Efficiencies and classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>j</td>
<td>CRS $\delta_j^l$</td>
</tr>
<tr>
<td>D1</td>
<td>0.8137</td>
</tr>
<tr>
<td>D2</td>
<td>1.0154</td>
</tr>
<tr>
<td>D3</td>
<td>0.8522</td>
</tr>
<tr>
<td>D4</td>
<td>1.0438</td>
</tr>
<tr>
<td>D5</td>
<td>0.8946</td>
</tr>
</tbody>
</table>

Tab.3 Interval efficiencies of the five projects using models (7) and (8)

<table>
<thead>
<tr>
<th>Projects</th>
<th>Efficiencies and classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>j</td>
<td>CRS $\delta_j^l$</td>
</tr>
<tr>
<td>D1</td>
<td>1</td>
</tr>
<tr>
<td>D2</td>
<td>1.006</td>
</tr>
<tr>
<td>D3</td>
<td>1</td>
</tr>
<tr>
<td>D4</td>
<td>1.0206</td>
</tr>
<tr>
<td>D5</td>
<td>1</td>
</tr>
</tbody>
</table>

For the super-efficiency DEA model with interval data, the upper and lower bound of relative efficiencies of each DMU under CRS and VRS are calculated by models (2) and (3), respectively, and the results are reported in Tab.2. For the SBM super-efficiency DEA model with interval data, the upper and lower bound of relative efficiencies of each DMU are calculated by models (7) and (8), respectively, and the results are reported in Tab.3.

(1) From Tab.2 and according to the class 1–3 in Section 2, DMU2 and DMU4 are classified in E++ under CRS condition due to $\delta_j^l=1.0154 > 1$ and $\delta_j^u=1.262 > 1$ and $\delta_j^l=1.0438 > 1$, $\delta_j^u=1.3304 > 1$. The other DMUs belong to E+. We can see that DMU2 and DMU4 are all evaluated to be DEA efficient. They together determine an efficiency frontier. These two projects are usually thought to be better than any other projects that are evaluated to be weakly-DEA efficient or non-DEA efficient. The classification of all DMUs is shown in the fourth columns of Tab.2. Similarly, under VRS condition, DMU2, DMU3 and DMU4 are classified in E++. The other DMUs belong to E+.

(2) By the same token, from Tab.3 and according to the class 4–6 in Section 4, DMU2 and DMU4 are classified in E++ under CRS condition and DMU3, DMU5 and DMU4 are also classified in E++ under VRS condition. The other DMUs belong to E+. Their performances are supposed to be more efficient than any other DMUs that are evaluated to be non-DEA efficient. The classification of all DMUs is shown in the fourth and eighth columns of Tab.3.

(3) In addition, we can see from the Tab.2 and Tab.3, DMU5 is infeasible by adopting super-efficiency model under VRS condition. However, the super-efficiency DEA model based on SBM is feasible under CRS or VRS condition, thus get around the fact the super-efficiency DEA model has been restricted to the situations where constant returns to scale. Furthermore, the SBM super-efficiency model can not only discriminate the performance of efficient DMUs from inefficient ones, but also distinguish between the efficient DMUs.

(4) According to the definition of possibility degree in Eqs. (9) and (10) and possibility degree matrix, taking the interval efficiencies under CRS condition in Tab.2 for example, we can obtain the following possibility degree matrix of paired comparison of these partially efficient DMUs:

$$P = \begin{bmatrix}
0.5 & 0 & 0.199 & 0 & 0.066 \\
1 & 0.5 & 0.8572 & 0.3368 & 0.4929 \\
0.801 & 0.1428 & 0.5 & 0.0803 & 0.2345 \\
1 & 0.6632 & 0.9197 & 0.5 & 0.5906 \\
0.934 & 0.5071 & 0.7655 & 0.4094 & 0.5
\end{bmatrix}$$
Then computing the sum of the element in matrix row by row, that is, $D_j = \sum_{i=1}^{n} P_{ij}$, we can get $D_1 = 0.765$; $D_2 = 3.1869$; $D_3 = 1.7586$; $D_4 = 3.6735$; $D_5 = 3.116$. Considering the classification and the ordering vector, the performances of interval DEA efficiencies of these projects are rated to be $\text{DMU}_4 > \text{DMU}_2 > \text{DMU}_3 > \text{DMU}_1 > \text{DMU}_5$. The results are shown in the fifth column of Tab.2, from which it can be seen obviously that the enterprise should give priority to project 4, which regard as the best enterprises’ project of independent innovation.

(5) From Tab.3, on one hand, we can see that $\text{DMU}_1$, $\text{DMU}_2$, and $\text{DMU}_3$ have the same lower bound of interval efficiency under CRS condition, so the order of these three DMUs is given by their upper bound, that is, $\text{DMU}_3 > \text{DMU}_1 > \text{DMU}_2$. Moreover, we compare and rank $\text{DMU}_2$ and $\text{DMU}_4$ by the possibility degree method. Through the computation, $P_{\text{DMU}_2, \text{DMU}_4} = 0.8377$, $P_{\text{DMU}_2, \text{DMU}_4} = 0.1623$, so $\text{DMU}_4 > \text{DMU}_2$. We get the ranking order of the five DMUs as $\text{DMU}_4 > \text{DMU}_2 > \text{DMU}_5 > \text{DMU}_3 > \text{DMU}_1$. The results are shown in the fifth column of Tab.3.

On the other hand, under VRS condition, $\text{DMU}_1$, $\text{DMU}_2$, and $\text{DMU}_3$ have the same lower bound of interval efficiency under CRS condition, so the order of these two DMUs is given by their upper bound, that is, $\text{DMU}_3 > \text{DMU}_1$. Moreover, we compare and rank $\text{DMU}_1$, $\text{DMU}_2$, and $\text{DMU}_4$ by the possibility degree method. Through the computation $P_{\text{DMU}_4, \text{DMU}_1} = 0.9451$, $P_{\text{DMU}_4, \text{DMU}_1} = 0.8247$, $P_{\text{DMU}_4, \text{DMU}_1} = 0.6587$, so $\text{DMU}_4 > \text{DMU}_2 > \text{DMU}_1$, that is, $\text{DMU}_4 > \text{DMU}_2 > \text{DMU}_3 > \text{DMU}_1$. The results are shown in the ninth column of Tab.3. In sum, the enterprise should give priority to project four as their best investment scheme of enterprise independent innovation project.

7 Conclusions

In this paper, we developed a super-efficiency model evaluated by slacks-based measurement with interval data in DEA for decision making units. As for IDEA, the upper and lower bounds for the interval efficiency scores of DMUs are individually obtained by the two transformed model with deterministic data. The results show that the SBM super-efficiency DEA approach is always feasible under the condition of CRS or VRS which is also proved in Du et al. (2010). In addition, this approach can get around the fact that the original DEA model can only distinguish the performance of efficient DMUs from inefficient ones, but cannot discriminate between the efficient DMUs. Furthermore, a possibility degree matrix composed by any two interval efficiencies is presented for comparing and ranking relative efficiencies of all DMUs. Last but not least, modeling DEA models with other types of imprecise data and further developing corresponding process to deal with it are important and practical issues for future research.

References


